SIMULATION-BASED ANALYSIS OF DEPENDENCY BETWEEN SALES VOLUME AND SERVICE LEVEL

Julija Petuhova and Yuri Merkuryev
Department of Modelling and Simulation
Riga Technical University
1 Kalku Street, Riga LV-1658, Latvia
E-mail: julija@itl.rtu.lv, merkur@itl.rtu.lv

KEYWORDS
Service level, market demand, inventory systems, hybrid simulation/analytical models, uncertain environment.

ABSTRACT

A traditional formulation of analytical inventory models does not consider a dependency between sales volume (market demand) and the customer service level provided. The additional assumptions make analytical solutions more complex. Inventory management under service dependent market demand is one of practical problems of that kind. The service dependent demand creates dynamic dependencies between itself and inventory management decisions. A simulation-based approach to managing inventory system under service dependent demand is elaborated in this paper. The appropriate hybrid simulation/analytical model has been developed for dealing with that problem. The modelling of inventory systems under service dependent demand is expanded by considering multi-period stohastic problems. Experimental studies conducted with the model are used to test the ability of traditional inventory systems to meet service level requirements in case of service dependent demand and to identify properties of such systems.

INTRODUCTION

As markets tend to be more and more customer-oriented, uncertainty connected with the end customer demand and its consequences in the supply chain have become an important subject of research (Merkuryev, Petuhova, 2001). Inventory control plays an important role in supply chain management. Inventory management involves balancing product availability, or a customer service, on the one hand, with the costs of providing a given level of product availability on the other hand. In fact, inventories exist only because supply and demand are not in harmony with each other. Inability to match product availability and market demand at a given time period leads to either stockouts or extra inventory carrying costs. If demand is unusually large, a stockout may occur. On the other hand, if demand is lower than anticipated, the replenishment arrives earlier than needed and inventory is carried. Managers have to balance these two types of risks. There are two extreme cases of what happens to a customer's order (market demand) when a product is temporarily out of stock (Silver, Pyke, and Peteron, 1998):

1. Complete backordering. Any demand, when out of stock, is backordered and filled as soon as an adequate-sized replenishment arrives. This situation corresponds to a captive market, common in government organizations (particularly the military) and the wholesale-retail link of some distribution systems (e.g., exclusive dealerships).
2. Complete lost sales. Any demand, when out of stock, is lost; the customer goes elsewhere to satisfy the need. This situation is most common at the retail-consumer link. For example, a person is unlikely to backorder a demand for a loaf of bread.

Stockouts are traditionally accounted for using the liner stockout cost. This stockout penalty, for instance, represents the costs associated with extra delivery charges for unavailable products. In other situations, it is used as a proxy variable representing loss of customer goodwill. However, this loss is very hard to measure. Schwartz, 1968 suggests that the loss of customer goodwill due to stockouts, especially in the retail industry, can be represented more accurately by considering a link between the service level provided and future demand (i.e., customers who face stockouts probably will switch to another retailer, while the high service level is likely to attract additional customers).
Determination of a market response on the provided customer service level is a complicated task (Gadžinskij, 2000). Market response curve, normally has the S-shaped form (see Figure 1). This can be explained by two reasons:

1. Most of markets require a minimal service level availability, that should be provided by a seller ("minimal service level barrier"). Activities before this minimal barrier will not give a sufficient profit, because the seller will not be perceived by a market. Operating before minimal barrier could be even unprofitable if customer service costs are higher than profit.
2. After reaching concrete customer service level, a market becomes non-sensitive to its further

increasing. Maximal service level barrier point on the market reaction curve indicates a service level after which a service increasing does not provide sales volume increasing.

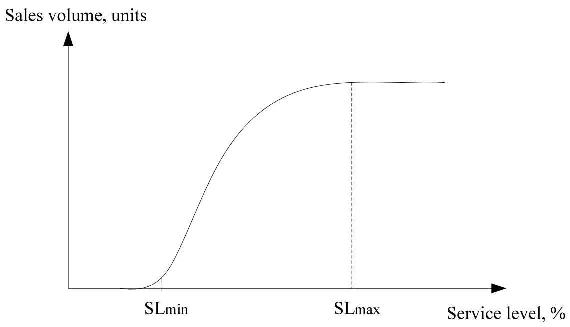

Figure1: Sales Volume Dependence on the Service Level

Clearly, the assumption of deterministic market demand is inappropriate in many distribution situations (Silver, Pyke, and Peteron, 1998). When demand is no longer assumed to be deterministic, shortage costs assume a much greater importance. The introduction of uncertainty in the demand pattern significantly complicates the inventory situation from a conceptual standpoint. Dependency between a sales volume and a service level implies that a market demand for further periods depends upon the service level achieved at the current period. A traditional formulation of analytical inventory models does not consider such dependence.

The existing research on inventory management with the dependency between sales volume and service level has been restricted to either situations with deterministic demand or two-period problems (Baker and Urban, 1988; Ernst and Powell, 1995). These limitations can be explained by an explicitly dynamic character of the problem leading to a complicated analytical analysis. Simulation modelling allows analysing a multi-period problem under stochastic demand. The market demand parameters change from one period to another in case of the multi-period problem. Such a behaviour can be observed in highly dynamic and competitive inventory systems. For instance, wholesalers of computer chips are often unable to meet the demand due to insufficient supplies from upstream supply chain levels. In case of the shortage, customers are likely to seek alternative vendors and may choose to place orders to a newly selected vendor for the following periods as long as the service level is maintained. The customers may switch back to the initial vendor if the newly selected vendor reduces its service level. Similar relationships between demand and performance of the inventory system are also observed in retail (Silver and Peterson, 1985). The research on inventory management with the dependency between market demand and service level also relates to research on inventory level dependent demand (see Chung (2003) for a recent account), delivery time dependent demand (see Ray and Jewkes (2003) for instance).

The multi-period inventory management problem with the dependency between sales volume (market demand) and provided service level, is considered. The service level dependent demand is modelled similarly to Ernst and Powell, 1995. It is updated according to the short-term service level observed during a simulation process. A simulation model is built around analytical models used for inventory management and updating demand parameters according to the observed short-term service level. That allows one to control inventory system behaviour and meet long-term service level requirements. The main objective of this research is to illustrate the difference in abilities of two types of inventory systems (traditional and with service dependent demand) to meet service level requirements.

The rest of the paper is organised as follows. In the next section, issues of inventory management under service dependent market demand are disscused. This is followed by simulation model description for evaluation of such inventory system. An explanation of an analytical model for service dependent market demand is given then. The last section is devoted to analysis of an inventory system behaviour providing experimental results. Finally, conclusions are provided.

INVENTORY SYSTEM

A single-item, single-stage, multi-period inventory system is considered. The traditional re-order point policy is used for inventory management. The order size is fixed independently of the re-order point level.

The service level is measured using a proportion of demand satisfied directly from the inventory. Any unsatisfied demand is lost. External demand is normally distributed. The parameters of the market demand (mean and standart deviation) change dynamically according to short-term fluctuations of the service level provided.

Relationships between the demand, service level and inventory parameters are shown in Figure 2.

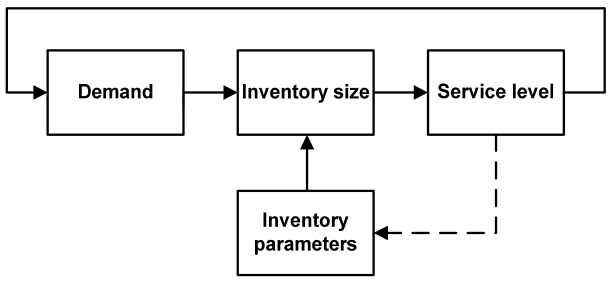

Figure 2: Interactions among Demand, Inventory Size and Service Level

The external (market) demand causes depletion of the inventory level. The inventory is replenished according to the re-order point policy specified by a set of the inventory control parameters (re-order point, order size, mean demand, demand standard deviation, lead time and target service level). The service level achieved during a relatively short time period is observed. This service level is most likely to differ from the target, required service level. In case of the dependency between the sales volume (market demand) and the service level, this causes changes in the demand parameters. A higher than target service level causes an increase in the mean demand and its standard deviation. A lower than target service level causes a decrease in the mean demand and its standard deviation. The increase in the demand parameters may result in a declining service level in forthcoming periods unless the inventory parameters are properly adjusted. The decrease in the demand parameters may cause overstocking unless the inventory parameters are properly adjusted. Therefore, a link representing the updating of the inventory parameters according to the observed short-term service level should be established.

SIMULATION MODEL

The inventory system described above has an explicitly dynamic character. Simulation is used to capture this behaviour of the system. The simulation model developed describes the inventory system and incorporates an analytical model for implementing a feedback between the simulated short-term fluctuations in the service level and the market demand parameters. The structure of the considered hybrid simulation/analytical model is given in Figure 3.

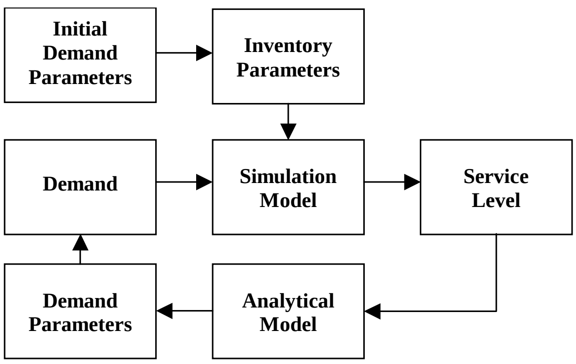

Figure 3: Structure of the Hybrid Simulation/Analytical Model

The comparative advantages and disadvantages of analytical versus simulation models are well known (for instance, see Nolan and Sorveign, 1972). Hybrid simulation/analytical models are used to attain some of the advantages of both types of models, while avoiding the disadvantages. A more detailed description of the considered hybrid simulation/analytical model can be found in Merkuryev et al., 2003.

Inventory management is based on the re-order point policy, where the order size Q is fixed independently of the re-order point level. The re-order point level is calculated using a formula:

$$ROP = LT * D_0 + z * \sqrt{LT} * \sigma_0 , \qquad (1)$$

where

LT – the replenishment lead time,
D_0 – an initial mean demand during one period,
σ_0 – initial standard deviation of the demand during one period,
z – a safety stock factor that depends on a specified target service level.

The simulation model is developed using the ARENA simulation modelling environment. Evaluation of the service level and updating of the demand parameters are implemented using the Visual Basic software.

ANALYTICAL MODEL FOR SERVICE DEPENDENT MARKET DEMAND

An impact of the provided service level on a future market demand is quantified similar to Ernst and Powell, 1995. In this approach, a linear relationship between the service level and the demand parameters is assumed. This dependence is evaluated based on the parameters estimated by experts. This means that the mean demand increases/decreases by α points if the change in the service level doesn't exceed a certain threshold:

$$\overline{D}_{t+1} = (1 + \alpha * (SL_t - SL_{t-1})) * \overline{D}_t \qquad (2)$$

where

SL_{t-1} – the short-term service level in the previous time period,
$\overline{D}_t$ – the mean demand of the current time period, α is a coefficient of the change in mean demand with increased/decreased service level.

The short-term service level is calculated each period using a formula:

$$SL_t = 1 - \frac{SO_t}{D_t} \qquad (3)$$

where

t – a current time period,
SO_t – an unsatisfied demand in period t,
D_t – an observed actual demand in period t.

The standard deviation of the demand for the new demand level is expressed as a function of the parameters α, β and the standard deviation σ_t of the current time period:

$$\sigma_{t+1} = \left[1 + \beta^2 \alpha (SL_t - SL_{t-1})\right]^{\frac{1}{2}} \sigma_t \qquad (4)$$

where

β – a coefficient of the change in standard deviation of demand with changed service level.

Cofficients α and β are specified based on experience and managerial subjective judgement (Ernst and Powell, 1995).

Here the linear relationship between the service level and the demand parameters is assumed. This dependence is evaluated based on the parameters estimated by experts. Non-linear relationships are possible in other situations when experts are able to estimate minimal/maximal service level barrier befor/after which market response becomes non-sensitive to its further decreasing/increasing.

In case if the decrease/increase in the service level exceeds a restricted constant, the mean demand is calculated by this formula:

$$\overline{D}_{t+1} = (1 + \alpha * (SL_t - SL_{t-1}) * MaxChange) * \overline{D}_t \quad (5)$$

where

MaxChange – a constant of the maximal change in the service level.

The standard deviation of the demand in this case is found by a formula:

$$\sigma_{t+1} = \left[1 + \beta^2 \alpha (SL_t - SL_{t-1}) * MaxChange\right]^{\frac{1}{2}} \sigma_t \quad (6)$$

The amplitude of changes in the demand parameters is restricted because the market is competitive and other players (not modelled here) will attempt to prevent any other players from expanding their market share substantially.

ANALYSIS OF INVENTORY SYSTEM BEHAVIOUR

The main objective of this research is to illustrate the difference in abilities of two types of inventory systems (traditional and with service dependent demand) to meet service level requirements.

Exsperimental design

The previous section discussed the approach to managing the inventory system in case of service dependent market demand. In this section, the behaviour of the inventory system under service dependent demand will be analysed and compared with the traditional one.

The objective of experimental studies is to determine, through simulation, the achieved long-term service level for the same safety stock factors in both inventory systems, the traditional and with service dependent demand.

A set of experiments with and without the feedback from simulation model to the analytical models, when the demand parameters are updated taking into consideration the observed short-term service level, is performed. Performance of the inventory systems is evaluated under different safety stock factor values (z) that vary in the range between -1.7 and 1.7. If order size is large, many customers have their demand satisfied before there is even an opportunity for a stockout. A negative value of safety stock factor is required to give a service level equal to the defined fill rate (target service level) (Silver, Pyke, and Peteron, 1998). Others factors of inventory management procedure such as initial market demand, its initial standart deviation, lead time, etc., are supposed to be constant values for all experiments.

The short-term service level (3) is observed each period in the inventory system with service dependent demand and the demand parameters are re-evaluated at the same time. The demand parameters re-evaluation is realised in accordance with the analytical model described above. In the traditional inventory system the demand parameters remain the same for the whole run. The values of the policy control parameters such as the re-order point and the order size are established once according to the inventory system parameters. While in the inventory system with service dependent demand they are established sistematically according to the changes in the demand parameters.

The provided long-term service level is the main performance measure. It is calculated at the end of each run by a formula:

$$SL_T = 1 - \frac{\sum_{t=1}^{T} SO_t}{\sum_{t=1}^{T} D_t} \quad (7)$$

where

T – a replication length.

The design consists of 8 experimental cells. The model was run for 5 replications. The length of each replication is defined as 250 weeks and a warm-up period is 20 weeks. Thus, simulation results are independent of the empty-and-idle intial state; there is no predetermined starting and finishing point for a simulation run.

Experimental results

Simulation results for one particular set of inventory system parameters are averaged over all replications and are reported in Table 1 and Table2. Table 1 represents the results for traditional inventory system and Table 2 for inventory system with service dependent demand.

In order to show how accurately the mean average of a long-term service level is estimated, confidence intervals for all experimental cells are calculated. A significant level (α) of 5% is selected. This gives a 95% probability that the value of the true mean lies within the confidence interval. The obtained intervals are rather narrow, so the estimation of the long-term service level is accurate. To indicate a precision of the experiments, average relative errors of a sample are calculated as well. The long-term service level (SL_T)

over 5 replications has the average relative error less than 2% for all experimental cells. This result indicates a perfect precision of the experiments.

Table 1: Simulation Results of the Traditional Inventory System over 5 Replications

z	Average SL_T	95% Confidence interval		
		Lower interval	Upper interval	Average relative error
-1.7	0.938	0.931	0.944	0.0024%
-1.5	0.944	0.936	0.951	0.0028%
-1	0.959	0.955	0.963	0.0008%
-0.5	0.974	0.972	0.977	0.0003%
0	0.987	0.985	0.988	0.0002%
0.5	0.993	0.991	0.995	0.0002%
1	0.997	0.995	0.999	0.0002%
1.7	0.999	0.998	1.000	0.0001%

Table 2: Simulation Results of the Inventory System with Service Dependent Demand over 5 Replications

z	Average SL_T	95% Confidence interval		
		Lower interval	Upper interval	Average relative error
-1.7	0.942	0.912	0.972	0.0467%
-1.5	0.959	0.933	0.984	0.0334%
-1	0.986	0.966	1.006	0.0196%
-0.5	0.997	0.994	0.999	0.0003%
0	0.999	0.998	0.999	0.0000%
0.5	0.999	0.999	1.000	0.0000%
1	1.000	0.999	1.000	0.0000%
1.7	1.000	1.000	1.000	0.0000%

Figure 4 illustrates the difference in the behaviour of the two systems. Based on the results achieved, it is possible to determine the value of the safety stock factor (z) needed to obtain a specified long-term service level in two types of the inventory systems: traditional and with service dependent demand.

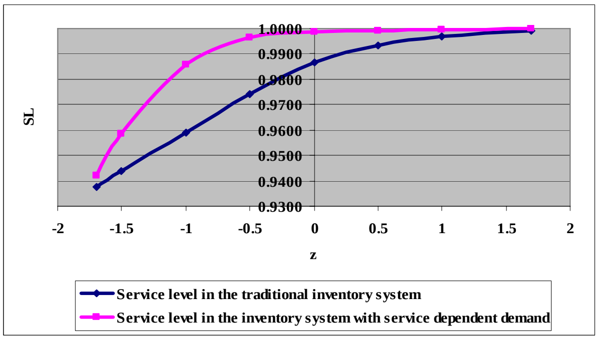

Figure 4: Long-term Service Level Dependence on the Safety Stock Factor

Setting the same safety stock factors, the traditional inventory system with the consatnt demand parameters gives a lower long-term service level than the inventory system with service dependent demand. That is, in the traditional inventory management system to achieve the specified long-term service level the higher safety stock level is needed than in case of the service dependent demand.

Safety stock requirements grow relatively quickly if the long-term service level required increases from 0.93 to 0.98. Then the long-term service level curve stabilises above 0.99. Characteristics of the service level versus safety stock factor depend upon particular values of the inventory system parameters, but in any cases there will be a point after which the long-term service level becomes not so sensitive to further safety stock factor increasing.

CONCLUSIONS

The simulation-based approach to managing inventory system under service dependent demand is developed. This approach integrates simulation and analytical models. The simulation model is used to capture inventory system dynamics, while incorporated analytical models are used to model market demand and to obtain inventory control decisions. The market demand parameters and control decisions are continuously updated according to demand changes in response to the short-term service level provided. The re-order point inventory system under service dependent demand has been expanded to multi-period, stohastic demand situation. The analysis of the inventory systems behaviour indicates that the inventory system with service dependent demand meets a higher long-term service level than the traditional inventory system with the same safety stock level. That is because in the inventory system with service dependent demand a link representing the updating of the inventory parameters according to the observed short-term service level is established and the control parameters are recalculated according to new values of the market demand parameters.

ACKNOWLEDGEMENTS

The present research was partly supported by the ECLIPS project of the European Commission "Extended collaborative integrated life cycle supply chain planning system".

This work has been partly supported by the European Social Fund within the National Programme “Support for the carrying out doctoral study programmes and post-doctoral researches” project “Support for the development of doctoral studies at Riga Technical University”.

REFERENCES

Baker R.C. and Urban T.L., 1988. A deterministic inventory system with an inventory-level-dependent demand rate. *Journal of the Operational Research Society*, 39 (9), 823-831.

Chung K.-J., 2003. An algorithm for an inventory model with inventory-level-dependent demand rate. *Computers & Operations Research*, 30 (9), 1311-1317.

Ernst R. and Powell S.G., 1995. Optimal inventory policies under service-sensitive demand. *European Journal of Operational Research*, 87, 316-327.

Gadžinskij A.M., 2000. Logistika. 3rd ed., Moskva, 2000.

Merkuryev Y., Petuhova J., and Grabis J., 2003. Analysis of dynamic properties of an inventory system with service-sensitive demand using simulation. *Proceedings of the 15th European Simulation Symposium "Simulation in Industry" (ESS'2003), October 26 - 29, Delft, The Netherlands*. Ed. by Alexander Verbraeck and Vlatka Hlupic. SCS-Europe BVBA, 2003. P. 509-514.

Merkuryev Y. and Petuhova J., 2001. Simulation of logistics systems: a survey. In: *Scientific Proceedings of Riga Technical University*, 5 (5), RTU 2001, 125-135.

Nolan R.L. and Sovereign M.G., 1972. A recursive optimization and simulation approach to analysis with an application to transportation systems. *Management Science*, 18 (12), 676-690.

Ray S. and Jewkes E.M., 2003. Customer lead time management when both demand and price are lead time sensitive. *European Journal of Operational Research*, 153 (3), 769-781.

Schwartz B.L. 1968. A new approach to stockout penalties. *Management Science*, 12 (12), 538-544.

Silver E.A. and Peterson R., 1985. Decision systems for inventory management and production planning. 2nd ed., New York: John Wiley & Sons, 1985.

Silver E.A., Pyke D.F., and Peterson R., 1998. Inventory management and production planning and scheduling. 3rd ed., New York: John Wiley & Sons, 1998.

BIOGRAPHIES

JULIJA PETUHOVA received her MSc Degree in Information Technology from Riga Technical University, Latvia in 2001. Currently she is a teaching assistant at Riga Technical University, Institute of Information Technology, Department of Modelling and Simulation. She continues her higher education towards getting a PhD in Information Technology. Her research interests include simulation methodology of logistics systems, supply chain dynamics, practical applications of simulation modelling, training and education via simulation-based business games.

YURI MERKURYEV is Habilitated Doctor of Engineering, Professor of the Institute of Information technology at Riga Technical University, Head of the Department of Modelling and Simulation. His professional interests include a methodology and practical implementation of discrete-event simulation, supply chain modelling and management, and education in the areas of modelling, simulation and logistics management. He is Programme Director of the Master-level curriculum "Industrial Logistics Management" at Riga Technical University. Prof Merkuryev has a wide experience in performing research and educational projects in the simulation area, at both national and European levels, and regularly participates in organising international conferences in the simulation area. He was General Chair of the 2005 European Conference on Modelling and Simulation held in June, 2005 in Riga, Latvia. Prof. Merkuryev has about 200 scientific publications, including 4 books. He is a Board member of the European Council of the Society for Modeling and Simulation International, President of the Latvian Simulation Society, and Board member of the Latvian Transport Development and Education Association.